APPLICATIONS OF LINEAR PROGRAMMING TO FACILITY MAINTENANCE PROBLEMS IN THE NAVY SHORE ESTABLISHMENT

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Submitted in partial fulfillment of
the requirements for the degree of

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IN
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ABSTRACT

Management science techniques have found many profitable uses in industrial applications, but their employment in the field of maintenance management has been lacking. An especially powerful tool of the management science discipline is that of linear programming. This technique is described and its several benefits discussed. Possible applications of the device are explored and a simple least-cost model developed. The model appears applicable to recurring facility maintenance work which may be accomplished by one of several methods involving various combinations of manpower, materials, machinery, and money. The theoretical model requires testing in a practical situation to verify its true value to the maintenance manager. If successful, however, it will be of limited value since it applies only to a portion of the maintenance work force at a Naval activity.

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I

STATEMENT OF THE PROBLEM

Introduction

A major problem facing the Naval officer or Navy civilian employee engaged in the management of the Navy's wide-spread shore establishment was pointed up in a recent issue of the Bureau of Yards and Docks' Weekly Report.[1] In a special article dealing with the pitfalls of financial management at a shore station Public Works Department, the Naval activity department responsible for facility maintenance, the critical need for a planned effort to reduce the backlog of essential maintenance was indicated. The article noted that a primary source of funds for the reduction must be diverted from savings on expenditures in other areas, including routine facility maintenance.

The existence of a large backlog of essential maintenance and repair projects is a critical problem and difficult to over emphasize. Projects included in the backlog are, in the judgement of experienced and qualified personnel, those which should have been accomplished in the past but have not been programmed because of

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[1] Bureau of Yards and Docks, Department of the Navy, "Rocks and Shoals of Public Works Department Financial Management," Civil Engineer Corps Weekly Report, Number 47-63 (November 21, 1963), p. 3.

the lack of dollar resources. The backlog, then, represents a rough measure of the condition of the Navy's real property investment. But while the backlog of essential maintenance and repair projects is a critical problem, it is a specific one. The method of attack, diversion of resources from other uses, illustrates a more general and traditional problem facing a Navy shore facility maintenance manager. This general issue, the production of the greatest possible benefits from scarce resources, has continued to receive constant attention since the first stirrings of economic interest. But there is a relatively new method of attack, a rapidly growing management device with which to cope with the problem. Known variously as operations analysis, operations research or, more recently, management science, this burgeoning discipline has as its central goal the application of scientific analytical methods to the solution of managerial problems. The device is reported to have genuine promise and the capability of providing management with more useful and complete information upon which to base decisions. It seems worthwhile, even imperative, therefore, to become familiar with its methods and possible applications to the problems of facility maintenance at Navy shore activities.

Although the analytical methodology currently known as management science probably had its beginnings in the

1920's with analysis of optimum production lot sizes and, later, of optimum sized inventories,[2] its first significant development occurred during World War II. Stemming largely from efforts already underway in Great Britain, operations research groups were organized first in this country in the Navy and Army Air Corps. The Navy's primary use of the techniques during the early part of the war was in analysis of anti-submarine warfare operations.

After a record of distinguished service during the war years, operations research techniques began to create the same stir of interest in civilian managerial circles as they had in the military. New methods of statistical analysis, mathematical tools such as linear programming and, above all, the success of the method during the war soon awakened civilian managers to its possibilities. Operations research groups are not uncommon today among the larger United States and British industries. Their use in military studies has grown from the first modest beginnings in World War II to their current level of pervasiveness in the Department of Defense and the military departments.

There were two significant factors present during

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[2] Charles C. Holt et. al., Planning Production Inventories and Work Force (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1960), pp. 8-9.

World War II which impelled the study and use of operations research techniques; both of them provide an insight into inherent characteristics of the discipline. These were the extremely weighty decisions required during combat operations and the availability of scientists released by the war effort from basic research laboratories.[3] The two factors reflect the keystones upon which the operations research method rests; the <u>scientific method</u> applied to <u>decision-making</u> problems. According to Morse, leader of the Navy's first operations research group:

> Operations research is a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control. [4]

In addition to the terms "scientific method" and "decisions", there is one more phrase in the above definition which merits particular attention - "quantitative basis." This term alone reveals significant aspects of the management science[5] discipline. First, that the

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[3] Philip M. Morse and George E. Kimball, <u>Methods of Operations Research</u> (first edition revised; (n.p.): The Technology Press of Massachusetts Institute of Technology and John Wiley & Sons, Inc., 1951), p.3.

[4] <u>Ibid</u>., p. 1.

[5] Because of its birth during World War II operations, the term "operations research" carries with it something of a connotation of strictly military analysis. Although nearly identical in meaning, "management science" will be used in this paper in order to draw attention to the application of the scientific method to managerial problems, particularly as applied to the "producer" side of the Department of the Navy.

methodology is primarily quantitative, rather than qualitative, in nature. Second, and this is a most important point, the quantitative analyses provide a "basis" for managerial decisions. In other words, management science techniques are <u>not</u> a replacement for sound managerial judgement and experience. Instead, the entire approach is to provide managers with soundly formulated data and information upon which to base decisions. Rather than seeking to undermine the manager's position, management science attempts to strengthen it by furnishing pertinent and valuable information not otherwise available. The objective of a leading professional society in the field, The Institute of Management Sciences, silouhettes this point quite clearly:

> The objects of the Institute shall be to identify, extend, and unify scientific knowledge that contributes to the understanding and practice of management. [6]

Management science, then, should be viewed as a powerful tool available to managers to use as any other analytical device; i.e., to provide information upon which to base

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[6] Thomas E Oberbeck, "TIMS-ORSA Cooperation," Selected Papers for Introductory Operations Analysis (unpublished collection of readings compiled and used at The United States Naval Postgraduate School, Monterey, California), citing The Institute of Management Sciences Bulletin, Special Issue (September 1, 1960), p. 2.

decisions. It is not a substitute for the decision-making process, nor does it relieve the manager of the responsibility for making decisions. Its true value, which is considerable, is its ability to provide information relevant to a problem in a manner at once penetrating, revealing, and normally, quantitative. Thus, the manager's ultimate decision more frequently may be based on data resulting from objective rather than subjective analysis. Management science may be said to multiply managerial talent, never to replace it, to permit the making of more accurate and cogent decisions.

Of specific interest to this paper is only one of the many tools of management science, that of mathematical or, more specifically, linear programming. Mathematical programming applies itself to the general economic problem of allocating scarce resources to various alternative uses, in order to obtain the greatest benefit from the resources. Robert Dorfman, a leading economist who has participated actively in the development of mathematical programming, describes the general situation in these terms:

> The central formal problem of economics is the problem of allocating scarce resources so as to maximize the attainment of some pre-determined objective. . . . Mathematical programming is based on a restatement of this same formal problem in a form which is designed to be useful in making practical decisions in business and economic affairs. [7]

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[7] Robert Dorfman, "Mathematical, or 'Linear' Programming: A Non-mathematical Exposition," *The American Economic Review*, XLIII (December, 1953), 797.

Linear programming, on the other hand, may be described as a special case of the more general technique of mathematical programming in that it deals with situations consisting of only straight-line, or linear, functions. Its primary objective, like mathematical programming, is planning the allocation of resources to alternative uses. As portrayed by three pioneers in the field, Charnes, Cooper, and Henderson, ". . . linear programming is concerned with the problem of planning a complex of interdependent activities in the best (optimal) fashion."[8] Thus, the technique is not one of execution, but a method of planning the use of resources in such a manner as to maximize the overall effort of an organization.

The applicability to the overall effort, as contrasted with efforts of individual components of an organization, makes linear programming particularly suited to the management level of a Naval activity's Public Works Officer or high echelons of the Navy facility maintenance organization. "One of the reasons why the programming tool has assumed importance", wrote George B. Dantzig, perhaps the leading United States authority on linear programming, "both in industry and in the military establishment, is that it is a method for studying the

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[8] A. Charnes, W. W. Cooper, and A. Henderson, An Introduction to Linear Programming (New York: John Wiley & Sons, Inc., 1953), p. 1.

behavior of systems."[9] In other words, the objective is to optimize the total effort of a system or organization. One of the reasons that linear programming has proven valuable in use has been its ability to avoid problems of sub-uptimization,[10] or letting one organizational component improve its operations at the expense of the overall goal of the entire organization.

The question now arises as to the general conditions of resource allocation planning under which linear programming is feasible. While the answer depends primarily on the particular aspects of a specific problem, there must be at least four stipulations present in a problem in order to use linear programming. First, and foremost, the variables which influence the attainment of a goal must be subject to quantification; i.e., susceptible to expression in numerical terms. Second, there must be alternative courses of action from which the manager must choose. Third, there must exist limitations or constraints on the feasible choices; e.g., a limitation on the number of men or materials available to perform a certain maintenance project. Further, the manager must be seeking to optimize some measure of

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[9] George B. Dantzig, "Thoughts on Linear Programming", Management Science, 3 (January, 1957), 131.

[10] Charles J. Hitch and Roland N. McKean, The Economics of Defense in the Nuclear Age (Cambridge, Mass,: Harvard University Press, 1961), pp. 128-131.

effectiveness or criteria, such as minimizing cost.[11] Finally, the various mathematical terms which express the relation-ships of the variables to the constraints and objective must be of the first power, or linear, straight-line relationships.[12] Linear programming may be applicable, then, in those situations in which a particular objective, expressed in linear mathematical expressions, is to be optimized through the application of available resources to one or more alternative courses of action, the resources being limited by some constraint.

While the linear programming technique may be an interesting mathematical exercise, the criterion of its value to the Public Works Officer and others connected with the maintenance management of the Navy's shore establishment is its ability to solve practical maintenance problems. Because it must meet the acid test of practicality to be of real worth, a resume of benefits of linear programming is in order.

Although the linear programming technique does provide optimal answers to problems of choosing from among alternative courses of action, this is by no means its

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[11] Kurt Meisels, A Primer of Linear Programming (New York: New York University Press, 1962), p. 4.

[12] While this condition is theoretically necessary, in actual practice non-linearity often may be approximated by linear functions with small loss in accuracy.

only managerial value. In fact, there are authorities who state unequivocally that this is not even the most important benefit. Gilbert and Undercuffler, for example, assert that, ". . . the real benefits of linear programming come not so much from the derivation of an optimum schedule as from the ability to determine other related production information."[13] An example of the related benefits is the cost, in terms of the output of the organization, of constraints on available resources. The constraints may be other than physical capacity restrictions, for example those imposed by certain policies and prodedures. The costs thus obtained are economic, or alternative costs, and are not available from traditional accounting procedures. They are of particular value to management, however, in that they measure the constraining restrictions in terms of the objectives sought. Still another benefit to be derived from linear programming is the relative ease by which various assumptions, restrictions, and variables may be analyzed for their effect on the overall objective. Through the use of a mathematical model, or set of equations describing the inter-relationships of variables, objectives, and constraints, various management choices may be altered and evaluated in terms of the objective.

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[13] Michael Gilbert and Edwin Undercuffler, Managematics: the Application of Operations Research (West Lafayette, Ind.: Management Perspective Series, 1959), p. IX-11.

The model also provides a means for determining the sensitivity of the objective to the variables in question, providing information as to which factors are truly significant.

Two additional benefits from linear programming are its ability to provide, almost force, a better management understanding of a particular problem, and its capability of permitting the so-called management by exception, or the shifting of a manager's attention from the technical details to a broader viewpoint. The first of these comes about as a result of framing the problem in terms of the mathematical model. If this is to be accomplished properly, it is essential that there be a complete understanding of the factors which directly or indirectly influence the organization's objective. Were there no other benefits to be obtained from the application of linear programming, increased comprehension alone could improve the quality of management decisions sufficiently to warrant its use. Once the system is framed in the linear programming format, however, the manager is free to assign computational details to a relatively unskilled employee and turn his attention to broader considerations of the organization's goals and policies. It is this last benefit, it appears, which has caused considerable misunderstanding of the objectives of management science. Because the methodology creates the opportunity for a

manager to divert his time from technical details, and places the routine in the hands of a subordinate, there is a tendency to assume that the manager has also divested himself of his decision-making prerogatives. The basis of this assumption, however, is the failure to realize that the manager has made the critical decisions concerning the operations under his control prior to and during the time that the mathematical model is being formulated. During this process the manager sets policies, objectives, and constraints, and from these doctrines a set of decision rules are formed which can be followed by a subordinate. Neither the subordinate nor the method can provide the critical answers and decisions which permit the decision rules to be formulated. This is the sole responsibility of the manager, and since his judgements form the basis for further action, it can hardly be asserted that he has lost his decision-making powers. Rather than abrogating his managerial responsibility, he has, in effect, shifted it to a higher plane.

In general, then, there are five primary areas in which the linear programming technique yields substantial benefits. The first is obvious - providing answers to the problem of allocating scarce resources to alternative courses of action in order to optimize an overall objective. The others, while less obvious, may actually be more valuable than the principle objective of the

technique. They include the beneficial knowledge of economic costs, in terms of output, which are imposed upon the organization by various limitations; the capability of analyzing assumptions, planning factors, constraints, and other factors to determine their effect on the overall objective; the improvement of management's understanding of the problem; and, the opportunity to practice management by exception, thus freeing valuable executive time and effort for pursuance of broader goals and opportunities.

In view of the significant benefits possible through the use of management science philosophies and techniques, it would seem likely that they are being utilized extensively in both military and industrial applications. Although becoming quite common in certain industrial production functions, as in the petroleum industry, there is an extreme lack of evidence to indicate even nominal use of management science in the field of facility maintenance. For example, a relatively recent survey of 157 different private companies was conducted to determine the current status of maintenance management in civilian applications. The results confirmed the fact that the "state of the art" of maintenance management was far behind the level of other managerial areas in the use of

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the latest management tools. A further example is given by an analysis of papers presented at the Twelfth Annual Plant Maintenance and Engineering Conference held in Chicago during the winter of 1960-61. A need for improved facility maintenance techniques in three major areas was generally expressed: (1) cost controls, (2) organization and methods, and (3) engineering techniques.[15] It is significant to note that there was no mention of management science principles in the report of the conference. The only conclusions which may be drawn from these examples are that either the new management tools are really not needed in industrial maintenance management, or else there is a general lack of awareness of their existence and/or potential benefits.

In the Navy, on the other hand, there does appear to be a greater recognition of the possibilities of the new methods. Construction specifications, for example, are more frequently requiring the use of the critical path method of scheduling, and there have been several studies instigated by the Bureau of Yards and Docks in the general area of management science as applied to maintenance management and engineering. Still, there have not been

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[14] George Frank, "What's Wrong With Maintenance Management Today," Factory, 120 (July, 1962), 72-78.

[15] "Maintenance Management Today: Troubled, Lazy," Factory, 119 (March, 1961), 76-77.

general applications of management science techniques at the field level, specifically Public Works Departments of shore activities.

The fact that management science procedures have not been generally applied to the maintenance management area raises the question of whether or not they actually should be used. Certainly, there are difficulties encountered in maintenance management which are not met in other managerial areas. "Problems confronting management regarding the study and control of the maintenance function," writes Raymond I. Reul, a nationally known industrial engineer and educator, "are quite different and far more complex than those encountered in production activities."[16] L. C. Morrow, another leading figure in the area of plant maintenance and engineering and editor emeritus of Factory magazine, has stated:

> It is shocking that many estimates in the field of maintenance show that we achieve only 40% of worker efficiency. This low figure is largely due to management short comings . . . 17

It would appear, then that the practice of maintenance management throughout the private industrial world is lagging behind other managerial areas. Plant maintenance activities appear to be in a position analagous to

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16 Raymond I. Reull; "Measuring and Controlling Costs," Chemical Engineering, 67 (September 19, 1960), 203.

17 "Maintenance Management Toady . . . ", op. cit., 76.

what certain authors have labeled as the second of three stages of an industrial firm's development. The first, both chronologically and in the level of sophistication, is the stage in which managerial decisions are made as the need for them arises. Judgemental factors are applied as criteria for decision-making as they seem necessary. Later, during the second stage, a formalized decision and control system is devised to ensure that the appropriate decisions are made, and that relevant data are considered. Finally, during the last stage of sophistication and growth, management efforts are concentrated in achieving "better" decisions and, eventually, "best" decisions; i.e., optimal decisions within a given framework.[18] With this growth and decision-making model as a basis, current maintenance management techniques within private industrial firms seem to fall squarely in the second stage. Navy practices, however, more closely fit the very beginnings of the third stage in that efforts are currently being expended, particularly in the Bureau of Yards and Docks, on attempts to find "better" methods of coping with the perplexing problems of maintenance management. Further, many years of experience with the controlled maintenance program, and its current level of sophistication, seem to qualify the Navy's facility

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[18] Holt, op. cit., pp. 6-7.

maintenance function for early graduation from the second stage of growth.

The question was earlier posed as to whether or not the management science approach should be applied to the maintenance management area. Because of the considerable complexities of maintenance management and the potential benefits which could result from more objective methods of obtaining information, it appears that management science techniques definitely should be used in the function of facility maintenance.

The management science approach to problem-solving and decision-making has developed extensively since World War II and now offers the modern day manager opportunities to rely less heavily on personal judgement and past experience. The newer techniques of management, of which linear programming is of particular interest in this paper, provide powerful tools of analyses which are of particular significance in attempting to find optimal solutions to problems. Efforts of the Bureau of Yards and Docks in attempting to discover "better" methods have been mentioned. One of these efforts was a study by the management consultant firm of Booz, Allen, and Hamilton in 1961. The findings contained in their report, emphasizing the need for a forward-looking maintenance management philosophy, indicate that increased effectiveness of Public Works Departments ". . . requires bridging

the gap between the present-day engineered programs and all of the ingredients essential for good management."[19] While complimenting the progress of maintenance management in the Navy to date, the consultants pointed out this signigicant observation:

> Clearly, the task ahead is of a different nature than the task which has been accomplished. A new approach is needed. [20]

It seems almost as clear that the "new approach" might well be that of management science.

The Problem

One of the managerial problems of the facility maintenance function at the Navy's shore activities is that of using scarce resources to the maximum advantage. Among management science philosophies and techniques linear programming is seen as a possible tool which might lend powerful assistance in solving this general problem.

Specifically, the question which this paper will attempt to answer is this. Theoretically, can the linear

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[19] Booz, Allen, and Hamilton, Management Consultants, Letter and report on Phase Two of Contract NBy-24971 for study, evaluation, and recommendations for increasing the effectiveness of the U. S. Navy's public works management program (1625 Eye Street, Washington, D. C., March 10, 1961), p. iii of the letter.

[20] Ibid.

[illegible]

programming technique be applied to problems of choosing from among alternative uses of scarce resources to optimize the facility maintenance function at Navy shore activities?

Definition of Key Terms

To avoid possible misunderstandings and to clearly differentiate between other connotations and the particular meanings used within this paper, the following definitions of significant words are given. Linear programming, first of all, is defined as a mathematical method of planning in which an objective is maximized or minimized while satisfying certain specified limitations or constraints. A constraint is interpreted as a limitation or condition which must be met in order to obtain a feasible solution to a problem. The number of man-hours of carpentry skills available per month could be given as an example of a practical constraint. Any problem solution which requires more than the maximum number of available man-hours is infeasible because it cannot be accomplished.

The objective function is the goal, expressed as a mathematical expression, which is to be optimized (maximized or minimized). Sometimes referred to as measure of effectiveness or criterion, an example of this term could be cost, in which case the objective would be minimization. In the case of commercial practices,

profit would be a very common criterion to be maximized. Opportunity cost has been selected as the measure of effectiveness for this paper. In this context, it is defined as the extra cost resulting from not performing a certain maintenance project when the condition requiring correction is first discovered. If it would cost "X" dollars to repair a certain road failure when first discovered, and "x+Y" dollars to repair the same failure at a later date, the opportunity cost is "Y" dollars.

Within the discussion which follows, the word maintenance is utilized as a generic term signifying any type of repair, replacement, restoration, overhaul, preservation, or other action taken either to prevent deterioration of facilities, correct an existing condition before it worsens or return an already deteriorated facility to useable condition. It thus encompasses the phrases "preventive maintenance", "repair", and "routine maintenance" normally used in the Navy. The term facility maintenance denotes work performed on machinery, grounds, utility lines, buildings, pavements, and other types of structures in order to differentiate it from the maintenance of aircraft, electronic equipment, vehicles, and other items of similar nature.

Assumptions

Because of the theoretical nature of this study, several assumptions are required in proceeding with the

investigation. First, it is assumed that the relationships among the numerous variables and objectives are, or may be closely approximated by, linear functions. This is not a difficult assumption for maintenance managers to accept with regard to constraining functions. A construction estimator, for example, implicitly makes this assumption when he determines cost per square yard or linear foot of road construction, or cost per square for roof replacements, and then proceeds to estimate the total cost by multiplying the unit cost by the total number of units. To accept this assumption for the measure of effectiveness, however, requires more confidence. It is as unlikely that the economic law of diminishing returns fails to operate in the facility maintenance function as in any other practical area. For example, if the maintenance of station roads is worth "X" utiles per square yard to the overall maintenance effort for the first 1000 square yards, it is difficult to accept the assumption that maintaining a million or billion square yards will be worth a million or a billion times "X" utiles. A more credible hypothesis would be that the maintenance of a billion square yards of roadway would be worth something less than a billion times "X" utiles, and perhaps considerably less. However, the assumption of linearity <u>within relevant ranges</u> normally under consideration, or that linearity may be <u>approximated</u>

within close tolerances is more acceptable because the conjecture of linearity is common to engineering sciences and arts. Therefore, the assumption is considered sufficiently rigorous for purposes of this paper and, indeed, for all practical applications of linear programming.

A second assumption seems equally plausible. This is the supposition that the payoff for performing a maintenance project now versus delaying it until a future date is the savings in opportunity cost. Because of aging of facilities, increasing costs for labor and materials (coupled with relatively stable funding levels), action of the natural elements, and other variables with time, the cost of a maintenance project normally increases if it is postponed. This is a common experience among maintenance managers and has led to fairly recent decisions from the Congress that a minimum amount of expenditures must be made on certain facility maintenance in order to decrease the opportunity costs.

A final assuntpion naturally and necessarily follows the second - that costs of proposed maintenance projects, both present and future, can be determined or estimated with reasonable accuracy. While there is little doubt that an average estimator can provide reasonably accurate cost estimates for many projects, some types of mainten-ance work are very difficult to estimate accurately.

However, even when accuracy is doubtful estimates are often the best information available. Regardless of whether an objective or a subjective analysis is utilized for evaluation, the most advantageous source of available information must be employed. Thus, should an estimate's accuracy be suspected, if it is the only available source of information it must be utilized as the "best". Although precision in an estimator's analysis of present or future maintenance costs cannot be demanded, fortunately it is not usually too much to expect reasonably accurate results. This assumption, ther fore, is consistent with practical situations and does not appear to prejudice the investigation of linear programming applications.

Limitations of the Study

In the process of attempting a particular study, practical considerations require setting certain boundaries to delineate a specific study area. While this requirement permits the isolation of a particular area of interest, it also tends to introduce certain limitations on the study and its conclusions. One of these limitations is the fact that this discussion is confined to the theoretical uses of linear programming. While practical experience and references have been probed, there have been no attempts to test the results and conclusions by actual application.

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Further, and partially over-lapping the first

limitation, it must be emphasized that this is not a complete management science study. An investigation of the physical sciences follows definite, logical, and time-proven procedures, and there is very little difference between the "scientific method" of classical physics and the management science approach to problem-solving. <u>This study, however, is merely an inquiry into possible uses of linear programming in maintaining the Navy's shore facilities; it is not a typically thorough management science approach into the problem of facility maintenance.</u> Unless this point is stressed there is danger of falsely presenting the very real and beneficial potential of management science as, according to Peter Drucker, a "gadget bag" for efficiency experts.[21] The discipline of management science, in other words, is not similar to a black box into which the analyst dips at random to find a tool to fit the rusty bolt he is trying to loosen. Rather than attempting to force a particular technique to fit a specific problem, management science seeks to apply the appropriate analytical methods where applicable. It formulates the problem into a certain framework, applies various analytical techniques to the solution, and designs controls to compare actual results

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[21] Peter F. Drucker, "Thinking Ahead", <u>[illegible] Business Review</u>, 37 (January-February, 1959), pp. 25-30, 146-150.

with those predicted. This study essentially by-passes the first and third steps and concentrates its attention on only one facet of the second.

A third limitation is the use of opportunity cost as a measure of effectiveness of the maintenance effort. Obviously, the final objective of a Navy shore activity is to furnish support to the operating forces. The function of facility maintenance, then, should be optimized <u>in relation to all other support furnished by other functions</u> at the shore station. In economic terms, the ratio of marginal productivity of the maintenance function to its marginal cost should, for optimization of the total station effort, be identical to the ratios of marginal productivities to marginal costs of all other functions. In engineering phraseology, optimization of overall station support occurs when the ratio of the benefit received from the last unit of facility maintenance utilized to the cost of this last unit equals the ratios of last unit returns to last unit costs of all other station functions. Since this point is amply demonstrated in any standard economics text, no proof will be presented here.[22] However, an elementary example of this economic principle may be shown through illustration of a hypothetical naval station consisting of three departments - Public Works,

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[22] See, for example, Paul A. Samuelson, <u>Economics, An Introductory Analysis</u> (sixth edition; New York; McGraw-Hill Book Company, 1964).

Supply, and Administration. Assume that the output from each of the departments can be measured in terms of fictitious units labelled "maints", "logs", and "admins", respectively. In compliance with the almost universal economic law of diminishing marginal returns with increasing output, after a certain point each unit of departmental output costs more than the preceeding unit. Thus, for example, as the Public Works Department increases output, the nine hundredth "maint" may cost $10.00, while the nine hundred and first "maint" costs $11.00. To maximize total station output, defined as units of "supports", within a given budgetary limit, it is necessary that the ratios of "supports" per last unit of departmental output to dollars per last unit of departmental output be equal. In mathematical expressions:

$$\frac{\text{supports/maint}}{\text{dollars/maint}} = \frac{\text{supports/log}}{\text{dollars/log}} = \frac{\text{supports/admin}}{\text{dollars/admin}}$$

Obviously, this optimization of station effort requires a precise quantitative measure of departmental output, and a means by which this output may be mathematically converted to equivalent units of station output. This is extremely difficult, if not impossible, because of numerous benefits derived from station output which do not lend themselves to generally accepted methods of quantification. The difficulty, then, forces consideration of analyses of departmental operations to attempt

to maximize departmental output within certain budgetary constraints. While this approach avoids the problem of quantifying overall station effort and, in some measure, that of converting departmental output to units of station effort, it does not provide relief from the formidable task of quantifying each department's output.

To illustrate the intricacy of departmental output measurement, consider only a portion of the Public Works Department's functions - maintenance. The quality of the maintenance effort is dependent upon many variables, a few of which are cost, reliability of equipment, ability of facilities to perform the designed purpose, even appearance. Because of the enormous difficulty in attempting to quantify many of the variables upon which the maintenance function depends, there is no currently valid and generally accepted quantitative measure of overall maintenance effort. Considering the fact that this is only one of several functions of the Public Works Department, the quantitative measurement of total departmental output is seen as an extremely difficult assignment.

Because of measurement difficulties, and to furnish a relatively simple device with which to illustrate possible uses of linear programming, an objective of opportunity cost has been chosen as a criterion for this study. <u>It must be emphasized, however, that opportunity cost is a grossly simplified and incomplete assessment of</u>

the facility maintenance function. It neglects elusive gains and losses which could be quantified with extensive study, such as the cost to the operating forces of inoperative generating equipment when electrical power is sorely required for a training or operational mission. The opportunity cost criterion also neglects variables which are more subjective than objective in nature. Management science and operations research analyses frequently encounter such obstacles to precise measurement. It is clear, however, that a decision-maker must consider various alternatives not only with respect to their quantitative nature but also in terms of what Hitch and McKean have chosen to label as "incommensurables",[23] factors for which there are no generally accepted objective methods of measurement. This problem of defining an all-inclusive measure of effectiveness is by no means unique, but it does serve to point up the previous observation that management science can never replace managerial judgement. Instead, analytical techniques should be viewed as sophisticated methods of providing more and better managerial information for more accurate decisions.

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[23] Hitch and McKean, op. cit., pp. 182-187.

II

A REVIEW OF SELECTED LITERATURE

Introduction

Plant maintenance costs assume relatively large proportions in the normal industrial firm's profit and loss statement. An average yearly expenditure of 6% of the original fixed capital investment is considered normal, although annual maintenance and repair costs may approach 20% of investment.[24] In 1957, for example, Factory magazine conducted a survey of 687 private companies by compiling data on file with the Securities Exchange Commission. From the total of over $7.5 billion spent on plant maintenance by the companies sampled, it was estimated that approximately $14 billion is expended annually by American industry to maintain its original plant investment of about $238 billion.[25]

Part of the investigation of this paper consisted of an attempt to discover practical applications of linear programming in the area of facility maintenance. Because of the huge expenditures made each year by American industry for plant maintenance, industrial

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[24] C. T. Zimmerman and Irvin Lavine, "Preliminary Cost Estimation - Investment and Operation," Cost Engineering, 7 (April 1962), pp. 4-18.

[25] "What Industry is Spending Today for Maintenance," Factory, 117 (February 1959), pp. 72-78.

[illegible]

trade magazines were considered a possible source of information. Accordingly, selected numbers of these periodicals were probed to uncover new trends in maintenance management, with the expectation that the linear programming technique would be included in the trends. In order to more thoroughly cover the subject, however, a further review of various periodicals of the management science and operations research disciplines was conducted to detect, in the general discussion of linear programming, possible applications in facility maintenance. Finally, as a check on the above two procedures, letters we addressed directly to twenty-three leading American firms representing the petroleum, chemical, steel, automotive, food, construction equipment, and general manufacturing industries. Each of the companies was asked for information concerning its use of modern management tools in the area of plant maintenance. Through these three approaches a fairly comprehensive survey was made of the possible industrial areas in which linear programming might be applied to maintenance management.

Results of the Review of Selected Literature

The search of industrial trade journals revealed essentially no information concerning the use of linear programming in the plant maintenance area. A significant number of articles dealt with the measurement of maintenance performance or effectiveness, however. One

Such measurement device is a method of assigning quantitative values to 215 various maintenance factors, grouped into nine major categories, through a management audit of the maintenance function. After analyzing and scoring each of the factors, various weights are assigned to the scores to arrive eventually at a numerical rating by category as well as a total rating for the entire maintenance function.[26] Another method, similar in that it expresses overall maintenance efficiency through eight "indicators", determines weighted performance percentages by comparing actual performance against set standards. The eight measurement factors are: (1) Indirect labor cost, (2) Other controllable expenses (power, fuel, materials, etc.), (3) Schedule effectiveness, (4) Cost reductions, (5) Downtime of machinery attributable to maintenance, (6) Call-in (overtime) hours, (7) Manufacturing losses attributable to maintenance, and (8) Backlog trend.[27] Still a third procedure, but of the same general nature, breaks the maintenance function into five categories and makes use

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[26] Thomas B. Foster, "Can You Measure Maintenance Performance?", Petroleum Refiner, 40 (January 1961), pp. 123-128.

[27] Ted Metaxas, "Measuring Overall Maintenance Efficiency," Mill & Factory, 69 (December 1961), pp. 84-87.

of a nomograph to determine weighted proficiency indices.

A somewhat different approach is based on the premise that over all maintenance effectiveness can be measured by comparing maintenance cost to plant value. In this method total annual maintenance cost, adjusted to a base year for monerary inflationary tendencies, is computed. Then the current replacement value of plant and equipment is compensated for rising prices by the Bureau of Labor Statistics Consumer Price Index. The final measure of maintenance "efficiency" is determined by dividing 100 times the adjusted maintenance cost by the adjusted plant replacement value.[29] Obviously, the lower the "efficiency" rating, which represents maintenance cost per $100.00 of plant value, the more efficient the maintenance function is presumed to be.

In addition to the current emphasis on measuring maintenance effectiveness, one other predominant topic, preventive maintenance, received substantial attention in industrial trade magazines. The practice of attempting to discover and arrest potential failures, particularly in mechanical and electrical equipment, preventive

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[28] Ralph M. Price, "Grade Your Performance With This One Page Score Sheet for Maintenance," Factory, 119 (September 1961), pp. 86-90.

[29] Geoffrey G. Corder and L. W. Saunderson, "2 New Ways to Measure Maintenance Efficiency," Factory, 118 (March 1960), pp. 98-100.

maintenance seems to have received more perceptive and objective analyses than the more general subject of maintenance performance. One typical approach is to define the total cost of preventive maintenance as a function of the direct cost of performing preventive maintenance inspections, cost of production losses due to machinery break-downs, and the cost of repairing failures when the occur. The objective is to reach a certain level of maintenance at which point the total preventive maintenance cost is a minimum. Graphically, the total cost function may be shown as indicated in Figure 1.[30]

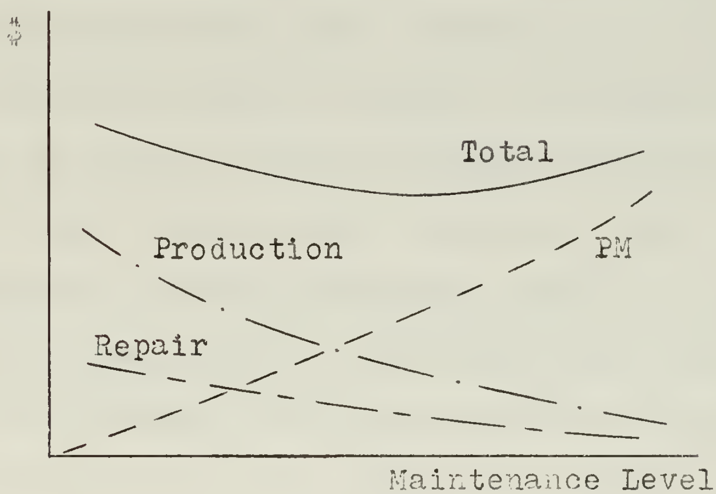

FIGURE 1

TOTAL PREVENTIVE MAINTENANCE COST

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[30] Jack F. Thornton, "Preventive Maintenance at Work," Paper presented at the 11th Annual Plant Maintenance and Engineering Conference, Philadelphia, Penna., 27 January 1960, Factory, 118 (April 1960), 115.

While the subject of linear programming is conspicuously absent from industrial trade magazines dealing with maintenance management, the management science literature abounds with practical applications as well as theoretical discussions. Although there are a wide diversity of situations to which linear programming can be applied, from agricultural through industrial to military spheres of interest, only a sample of the information available in the literature will be presented to show something of the range of applications.

Partly because systematic procedures for solving the linear programming problem were unknown prior to 1947,[31] the first industrial use of linear programming did not take place until 1953. Previously, SKF Industries had been using Gantt chart techniques to control the manufacture of rings, rolling elements, and miscellanious parts for anti-friction bearings. With the installation of a linear programming system in January 1953, however, the impression that the Gantt chart procedure had produced efficient results was shattered. After installing the linear programming technique it was discovered that about 6000 machine-hours per month in excess of actual needs were being used. Within six months this excess capacity was reduced to 1500 machine-hours, and in a total of two years only 400-600 excess

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[31] Dantzig, op. cit., 137.

machine-hours per month remained. Linear programming not only decreased costs by 11%, but it increased the production schedule actually met from 90% to nearly 100%, and fixed responsibilities for failures to meet schedules. Indirect benefits attributed to linear programming were increased knowledge of specific operator and machine limitations and material problems which had caused off-standard production.[32]

The petroleum industry first became aware of linear programming techniques in the period of 1952-1954,[33] and has continued to lead American industrial use of the tool since that time. Many different applications have been made, including optimum blends of petroleum constituents to produce various products for maximum profits. In Great Britain, on the other hand, linear programming has been applied to various situations in

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[32] George Morin, "More Effective Planning With Linear Programming," Linear Programming and Inventory Management Seminar, Proceedings of seminar sponsored by Methods Engineering Council, Management Consultants, New York and Pittsburgh, September 15-16, 1955, pp. F-1-F-2.

[33] W. W. Garvin et. al., "Applications of Linear Programming in the Oil Industry," Management Science, 3 (July 1957), 407.

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the mining[34] and the steel industries.[35] The conclusions of a study conducted in 1961 on the use of linear programming in three relatively small firms typifies the general attitude concerning the benefits of the procedure. Although none of the companies investigated was large, linear programming was proved to be a valuable tool for each of them. An interesting conclusion was that, in the long run, knowledge of the economic costs, sometimes called "shadow prices", is of greater value than the answer to the direct allocation problem.[36]

Two other uses of linear programming, while outside the industrial area, are of interest in demonstrating the range of possible applications of the method. The first of these is the use of the technique in planning an optimal mix of offensive and defensive missile and aircraft systems, subject to budgetary, aircraft carrier deck space, and other constraints.[37] The second, taken

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[34] K. B. Williams and K. B. Haley, "A Practical Application of Linear Programming in the Mining Industry," Operational Research Quarterly, 10 (September 1959), pp. 131-137.

[35] J. R. Lawrence and A. D. J. Flowerdew, "Economic Models for Production Planning," Operational Research Quarterly, 14 (March 1963), pp. 11-29.

[36] Ernest Koenigsberg, "Some Industrial Applications of Linear Programming," Operational Research Quarterly, 12 (June 1961), pp. 105-114.

[37] Bernard S. Albert, "Cost-effectiveness Evaluation for Mixes of Naval Air Weapons Systems," Operations Research, 11 (March-April 1963), pp. 173 ff.

from an engineering journal, indicates the use of apecialized cases of the general linear programming technique, particularly the so-called "transportation" problem, in the construction industry.[38]

Despite the interesting aspects of the various uses of linear programming, only brief resumes of the examples from the literature have been presented because none of them describes the exercise of the method in the area of maintenance management. Only a single, very recent study was discovered in the literature selected for review in which linear programming was considered for use in facility maintenance problems. A theoretical exploration of the possibility of employing integer linear programming models (those in which the coefficient of the various variables are prevented from taking fractional values) in scheduling preventive maintenance, the study attempted to meet the objective of timely accomplishment of the maintenance effort with minimum fluctuation in manpower required at any one time. The authors concluded that integer programming is not yet sufficiently sophisticated for successful use in the accomplishment of the stated objective.[39]

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[38] Robert M. Nicholls, "Operations Research in Construction Planning," Journal of the Construction Division, Proceedings of the American Society of Civil Engineers, no. CO2 (September 1963), pp. 59-74.

[39] Harvey M. Wagner, Richard J. Giglio, and R. George Glaser, "Preventive Maintenance Scheduling by Mathematical Programming," Management Science, 10 (January 1964), pp. 316-334.

Evidence of uses of linear programming in the area of facility maintenance was lacking in the literature, and replies received from leading American firms provided no further information along these lines. As was evident from the trade journal survey, maintenance management in the companies from which answers were received was primarily interested in the measurement of maintenance performance and preventive maintenance, and various institutional arrangements to obtain better cost information and work control. In general, the review of selected literature yielded only negative results. It appears that applications of linear programming to the maintenance management function are as yet undiscovered.

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THE STUDY

The Linear Programming Technique

Probably the first work on the central theory of linear programming was written by Kantorovich of the Soviet Union as early as 1939.[40] The full value of the method, however, was never fully exploited until a logical computational algorithm, or mathematical method of solution, was devised in 1947 by this country's leading authority, George B. Dantzig.[41] With the advent of the simplex method, the full force of the tool was soon realized in attaining the objective of linear programming, that of:

> ". . . determining a program of activity by finding the optimum solution of a group of restrictive linear equations." 42

An explanation of the simplex method, the most general computational technique of linear programming, can best be opened with a simple illustrative example. With only two variables involved in this example, the general theory of the solution may be shown graphically.

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40 L. V. Kantorovich, "Mathematical Methods of Organizing and Planning Production," Management Science, 6 (July 1960), pp. 363-422.

41 George B. Dantzig, "Maximization of a Linear Function of Variables Subject to Linear Inequalities," Activity Analysis of Production and Allocation, Cowles Commission for Research in Economics Monograph No. 13, Tjalling C. Koopmans, editor (New York: John Wiley & Sons, Inc., 1951), pp. 339-347.

42 Dakota Ulrich Greenwald, Linear Programming (New York: The Ronald Press Company, 1957), p. 3.

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Assume that a small cabinet-making shop produces only two types of furniture, tables and desks. Involved in this simplified version of furniture manufacturing are three processes, cutting the lumber into required forms and dimensions, assembly of the parts into completed pieces of furniture, and finishing the surfaces of the pieces. Because of capacity restrictions, each of the three processes can produce specific maximum amounts of either product. For example, if 100% of the cutting operation were devoted to tables, sufficient material for a maximum of 40 tables per month could be pre-cut. If all the capacity were devoted to desks, however, parts for a maximum of 70 desks per month could be cut. The total capacity may be divided between the two types of furniture, of course; 50% devoted to each product would produce material for 20 tables and 35 desks. Similarly, capacity restrictions on the assembly operation permit production of a maximum of 65 tables or 45 desks per month, and for the finishing stage the maximum production is 50 tables or 50 desks per month. Assume further that the profit per table when it is sold is $12.50 and the profit per desk is $10.00, and that as many desks and tables can be marketed as are produced. The problem is to find the particular production combination of tables and desks to maximize profits.

The immediate intuitive solution to the problem might be to produce nothing but tables since there is 25% more profit per table than per desk. The reason that

this solution is incorrect, however, may be determined by reference to Figure 2.

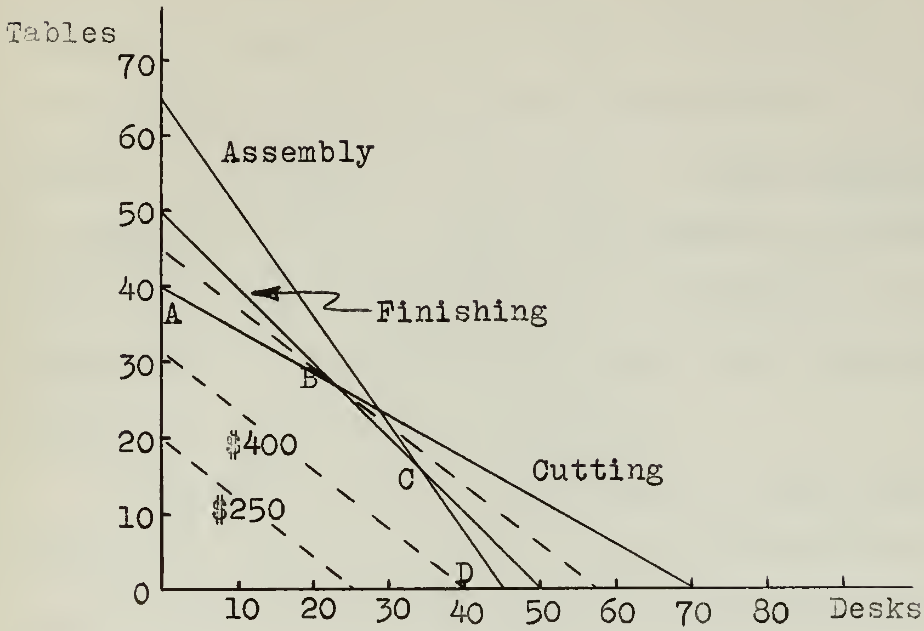

FIGURE 2

OPTIMAL PRODUCTION POINT

The linear constraining functions are shown for each of the production operations. The polygon OABCD, formed by the three constraints and the two axes, is known as the "convex hull", and contains all feasible production possibilities within its boundaries. For example, it is feasible, although not sensible, to produce 20 tables and 10 desks, or 15 tables and 20 desks. Further, all points lying directly on the boundaries of the convex hull are also feasible solutions to the problem. An example of one such point is 40 tables and zero desks.

The objective in this example is to maximize profits. Shown as dashed lines in Figure 2 are several values of the objective function. These parallel lines represent

total profits available from selling the amounts of either tables or chairs at the intersection of the applicable axis. For example; refer to that value of the objective function labled "$250." If 20 tables were sold, the resulting profit would be 20x$12.50 for a total of $250.00. Similarly, if 25 desks, or <u>any combination of tables and desks on the $250 objective function</u> were sold, the resulting profit would be $250.00. As the distance of the objective functions from the origin increases, total profit increases.

Obviously, then, the solution to this simple illustration is seen to be that feasible combination of tables and desks chosen so that profits are maximized. There is but one point in Figure 2 which meets all capacity restrictions with maximum profits; point B is called the optimal feasible solution.[43] Maximum attainable profits then, are $567.50 per month at a production level of 27 tables and 23 desks.

Several interesting relationships in Figure 2 may be pointed out. First, the assembly operation is not used to capacity at the optimal production level. It is common to refer to this excess capacity as "slack". Next, it is seen that, if there were additional finishing capacity, the objective function could move to the intersection of the assembly and cutting constraints.

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[43] Kenneth E. Boulding and W. Allen Spivey, <u>Linear Programming and the Theory of the Firm</u> (New York: The Macmillan Company, 1960), p. 63.

That point represents a total of 24 tables and 29 desks for a return of $590.00 per month in profits. Because the finishing capacity is limited, however, the cabinet-maker is <u>losing</u> $590.00 minus $567.50, or $22.50 per month. This is the opportunity cost of the finishing capacity restriction. If the opportunity cost were divided by the number of units of input (such as extra man-hours) required to appropriately increase the capacity, the cabinet-maker would then know the unit economic cost, or "shadow price", of the resources in the finishing operation. This valuable piece of managerial information cannot be obtained by traditional accounting procedures.

Although the simple illustration is of trivial consequence, it does exemplify the basic theory of linear programming methodology. With large numbers of variables, of course, it becomes conceptually impossible to employ geometric solutions to the problem. But whether there are two or two thousand variables, the basic idea is the same. This illustration showed an example of maximization, but the geometric intrepretation could be equally applied to a problem of minimization.

In a situation consisting of many variables, the algebraic solution must be employed. The most general form of the computational technique, the simplex method, consists of several definite procedural steps.[44]

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[44] Edward H. Bowman and Robert B. Fetter, Analysis for Production Management (revised edition; Homewood, Illinois: Richard D. Irwin, Inc., 1961), pp. 110-111.

1. The problem is framed by stating all requirements and slacks explicitly, determining all applicable costs and revenues, determining the objective function, and establishing the computational framework.

2. An initial solution, which must be technically feasible though it may be economically absurd is determined.

3. Alternative choices are evaluated as to the total effect of an incremental change on the objective.

4. The most favorable alternative is chosen and the number of units of this alternative to be brought into the solution is determined.

5. A new solution is formed encompassing the information determined in step 4.

6. Since the process is iterative, steps 3 through 5 are repeated until there are no longer any favorable alternatives to be brought into the solution.

The first step consists of properly framing the problem in mathematical language by expressing the constraints and objective function in linear functions. Consider, for example, the two product cabinet-making shop discussed earlier. Assuming 20 shop-days per month as the available capacity of each operation, the number of shop-days required for each unit of product may be computed by dividing shop-days per month by the number of units per month.

	Cutting	Assembly	Finishing
Tables	20/40	20/65	20/50
Desks	20/70	20/45	20/50

The three constraints may now be placed in model form by letting P_t stand for the number of tables and P_d for the number of desks to be made during the month. Since the capacity of each operation cannot be exceeded, although less than full capacity might be utilized, we have:

(1) $20/40\ P_t + 20/70\ P_d \leq 20$ shop-days (Cutting)
$20/65\ P_t + 20/45\ P_d \leq 20$ shop-days (Assembly)
$20/50\ P_t + 20/50\ P_d \leq 20$ shop-days (Finishing)

Since inequalities are not permitted in the simplex procedure, let P_1, P_2, and P_3 stand for shop-days of slack time for the cutting, assembly, and finishing operations, respectively. Equations (1) then become:

(2) $20/40\ P_t + 20/70\ P_d + P_1 = 20$
$20/65\ P_t + 20/45\ P_d + P_2 = 20$
$20/50\ P_t + 20/50\ P_d + P_3 = 20$

To determine the objective function, Z, recall that each table returns \$12.50 and each desk \$10.00 in profit. Since the numerically subscripted slack variables yield zero profit, the objective function to be maximized may be stated as:

(3) $Z = (12.50)\ P_t + (10.00)\ P_d + (0)\ P_1 + (0)\ P_2 + (0)\ P_3.$

The problem has now been framed in the appropriate linear programming model with the three constraints, equations (2), and the objective function equation (3).

The format of the general linear programming problem has been described by McCrae in the following manner:

> . . . let a_{ij}, b_i, and c_j be sets of constraints ($i = 1, . . ., m$; $j = 1, . . ., n$) and x_j ($j = 1, . . ., n$) be a set of variables. We seek solutions $x = (x_1, x_2, . . ., x_n)$ which satisfy the inequalities
>
> $$\sum_{j=1}^{n} a_{ij}x_j \leq b_i, \quad i = 1, . . ., m;$$
>
> $$x_j \geq 0, \quad j = 1, . . ., n;$$
>
> and at the same time maximize the linear functional
>
> $$Z = \sum_{j=1}^{n} c_j x_j$$ [45]

In more expanded form, the general problem is to maximize the objective function:

$$Z = C_1X_1 + C_2X_2 + C_3X_3 + . . . + C_nX_n,$$

subject to the constraints:

$$A_{11}X_1 + A_{12}X_2 + A_{13}X_3 + . . . + A_{1n}X_n \leq B_1$$

$$A_{21}X_1 + A_{22}X_2 + A_{23}X_3 + . . . + A_{2n}X_n \leq B_2$$

$$. . . . .$$

$$A_{m1}X_1 + A_{m2}X_2 + A_{m3}X_3 + . . . + A_{mn}X_n \leq B_m$$

and $X_i \geq 0$, $i = 1,2, . . ., n$. (negative production is not possible). As before, A_{ij}, B_i, and C_j are known constants.

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[45] Vincent M. McCrae, "Linear Programming," Operations Research and Systems Engineering, Charles D. Flagle, William H. Huggins, and Robert H. Roy, editors (Baltimore: The Johns Hopkins Press, 1960), p. 367.

The following expanded version of the cabinet-making shop is presented to illustrate the computational procedures of the simplex method. Assume that the same three operations, cutting, assembly and finishing, are required but that the product line has been expanded to include chairs and bookcases as well as tables and desks. Required numbers of man-hours, the profits per unit, and the number of available man-hours per month in the three operations are given as:

	Cutting	Assembly	Finishing	Profits
Bookcases	3	5	4	$ 6
Chairs	4	2	3	5
Desks	8	12	4	10
Tables	6	8	6	12
Man-hours	160	200	120	--

Letting P_b, P_c, P_d and P_t stand for units of bookcases, chairs, desks, and tables produced each month, and P_1, P_2, and P_3 for possible slack time in the cutting, assembly, and finishing operations, respectively, the problem may be stated in the general format. Subject to the constraints:

(4) $3P_b + 4P_c + 8P_d + 6P_t + P_1 = 160$

$5P_b + 2P_c + 12P_d + 8P_t + P_2 = 200$

$4P_b + 3P_c + 4P_d + 6P_t + P_3 = 120$, and

(5) $P_b, P_c, P_d, P_t \geq 0$,

the objective function to be maximized is:

(6) $Z = 6P_b + 5P_c + 10P_d + 12P_t + (0)P_1 + (0)P_2 + (0)P_3$

Having framed the problem, the next step is to formulate an initial solution. Since this solution must be technically feasible but not necessarily economically logical, the most convenient solution with which to begin the simplex computations is that in which all except the slack variables are set equal to zero. In effect, the initial solution is to do nothing, or consume the entire capacity of each operation with slack time.

It is most convenient at this point to place the problem in the form of a simplex tableau and perform the computations within the tableau. As can be seen in Figure 3, the variables are listed at the top of the tableau, above which are indicated the corresponding unit profits. In the main body of the matrix are the variable coefficients of the set of constraining linear equations of the mathematical model. The base $(Z_j - C_j)$ row to the right of the double line indicates the opportunity cost per unit to the overall objective for <u>not</u> having the variable in the solution. For example, in Figure 3, the "-6" in the base row under the P_b column shows that the cabinetmaker is losing $6.00 per bookcase in profits because there are no bookcases in the solution. The figure in the base row under the P_o column shows the total profit.

FIGURE 3

INITIAL SOLUTION

ILLUSTRATIVE SIMPLEX METHOD

c_j \ z_j			0	0	0	6	5	10	12	
		P_0	P_1	P_2	P_3	P_b	P_c	P_d	P_t	
0	P_1	160	1	0	0	3	4	8	6	$\frac{160}{6} = 26.7$
0	P_2	200	0	1	0	5	2	12	8	$\frac{200}{8} = 25.0$
0	P_3	120	0	0	1	4	3	4	6	$\frac{120}{6} = 20.0$ **
$z_j - c_j$		0	0	0	0	-6	-5	-10	-12	
									**	

The second and third columns indicate the names and amounts of the variables in the solution; the first column repeats their profit contributions. In the first tableau, the initial solution is seen to be that in which the total capacity of each operation is taken up by slack, with a net profit of zero.

As an illustration of the identity of the first simplex tableau to the set of constraints, consider the first of equations (4). By rearrangement of terms, this becomes:

(7) $160 = (1)P_1 + 3P_b + 4P_c + 8P_d + 6P_t$,

[illegible]

[illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

[illegible]

[illegible]

$$[illegible]$$

which is identical to:

$$(8) \quad 160 = (1)P_1 + (0)P_2 + (0)P_3 + 3P_b + 4P_c + 8P_d + 6P_t$$

The similarity between equation (8) and the P_1 row of the first tableau is immediately obvious.

Having framed the problem and determined a feasible solution, the next step is to alter the solution to include the most favorable alternative. With the sign convention employed in this method of simplex computations, the most favorable choice is that variable with the largest negative number in the base row. If the problem were one of minimizing an objective function instead of maximizing the best alternative would be that with the largest positive number in the base row. Since the objective in this case is to maximize profits, P_t, with a base row figure of -12, is obviously the most favorable alternative. The number of tables which may now be included in the solution is determined by analysis of the coefficients of P_t and the capacity limitations. For each unit of P_t brought into the solution it is necessary to remove six units of cutting slack, P_1, eight of assembly slack, P_2, and six units of finishing slack, P_3. The reason for this is clear when it is recalled that six, eight, and six man-hours of cutting assembly, and finishing, respectively, are required for one table. The maximum amount of P_t which can be entered into the solution is given by the minimum, non-negative number resulting from

dividing the P_o column figures by the P_t column figures, as shown to the right of the first tableau. The algorithmic technique for entering 20 tables in the solution is to "pivot" the column P_t about the "index number" at the intersection of the "key column" and "key row", thus "replacing" the row P_3 with the column P_t. When 0, P_3, and 120 in the first three columns are thus replaced by 12, P_t, and 20, the remaining procedure in setting up the second tableau and commencing the iterative process again consists of changing the coefficients of the first tableau to correspond to the new situation. This is accomplished first with the key row figures by dividing each of them by the index number in the same manner as 120 was divided by the index number, six, to determine the permissible number of units of P_t to enter into the solution. After finding the new P_t row coefficients, the remaining variables in the solution, P_1 and P_2, and their profit contributions are inserted in the proper columns of the second tableau. The variable names and their individual profit contributions at the top of the matrix remain unchanged. All other coefficients are then altered to compensate for the new situation through the following rule: the new value of a particular coefficient equals the old value minus the old value of the corresponding key row number times the old value of the corresponding key column number divided by the index number. The old (first tableau) value of the coefficient in the P_2 row and P_c column is "2". The

FIGURE 4

SECOND TABLEAU

ILLUSTRATIVE SIMPLEX METHOD

c_j \ z_j			0	0	0	6	5	10	12		
		P_0	P_1	P_2	P_3	P_b	P_c	P_d	P_t		
0	P_1	40	1	0	-1	-1	1	4	0	$\frac{40}{4} = 10.0$	
0	P_2	40	0	1	-1.3	-.3	-2	6.67	0	$\frac{40}{6.67} = 6.0$	**
12	P_t	20	0	0	.17	.67	.50	.67	1	$\frac{20}{.67} = 30.0$	
$z_j - c_j$		240	0 0	0	2	2	1	-2	0		
								**			

corresponding Key row value is "3", the corresponding key column number is "8", and the index number is "6". Thus the new (second tableau) coefficient for the P_2 row and P_c column is 2-(1/6)(3x8) = -2. In this case the negative sign indicates that bringing one unit of P_c into the solution requires adding, or "negatively removing," two units of P_2. The second tableau of the solution is shown in Figure 4.

The same procedural steps are repeated until all values in the base (Z_j-C_j) row are either positive or zero. When this condition is reached, as in the third

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

FIGURE 5

THIRD TABLEAU

ILLUSTRATIVE SIMPLEX METHOD

c_j \ z_j			0	0	0	6	5	10	12	
		P_0	P_1	P_2	P_3	P_b	P_c	P_d	P_t	
0	P_1	16	1	-.6	-.2	-.8	2.2	0	0	
10	P_d	6	0	.15	-.2	-.05	-.3	1	0	
12	P_t	16	0	-.1	.3	.7	.7	0	1	
$z_j - c_j$		252	0	.3	1.6	1.9	.4	0	0	

tableau of Figure 5, the optimum solution to the problem has been reached. To return a maximum profit of $252, the cabinet-maker should produce 16 tables and 6 desks per month, with excess cutting capacity of 16 man-hours.

Of particular managerial interest are the economic costs of constraints. These opportunity costs are known by various names, including implicit values, shadow prices, and, despite the fact that normal bookkeeping methods cannot furnish them, even accounting costs or prices. In the final solution of the iterative process they are shown in the base row under the slack variable correspond-

[illegible]

[illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

ing to the particular constraint. Since there is excess capacity in the cutting operation, its opportunity cost is zero; it corresponds to an economic "free resource". The assembly operation, however, has an implicit value of \$.30 per man-hour and the finishing operation is "worth" \$1.60 per man-hour. To illustrate that these figures actually do measure the value of the constraints, observe that if they are multiplied by the number of units available in the applicable operation,

$$(.30)(200) + (1.60)(120) = 60 + 192 = 252,$$

the resulting answer is identical to the optimal profit figure. Thus, if the cabinet-maker could somehow decrease his capacity in the cutting operation and increase his finishing capacity, he would shift his resources from an alternative which is worth nothing to him, to one which is valued at \$1.60 per unit of capacity in terms of profit. This is a conspicuously significant piece of managerial information.

Possible applications of linear programming

Almost without exception, every authority or writer in the field of linear programming develops a unique list of conditions necesary for application of the technique. Robert Dorfman, for example, indicates certain theoretical conditions,[46] some of which may be circumvented in the

[46] Robert Dorfman, Application of Linear Programming to the Theory of the Firm (Berkeley: University of California Press, 1950), pp. 32-34.

practical situation. A more pragmatic check list is furnished by management consultant Donald Moffett. Linear Programming is applicable, he indicates, when: (1) the problem consists of coordinating a number of related and interdependent decisions, (2) a number of alternative choices exist, (3) the various conditions of the problem may be expressed mathematically, (4) a single objective exists which may be xpressed mathematically in terms of individual activities within the problem, and (5) pertinent data to express the mathematical relationships is available or may be estimated.[47]

Still another list of practical criteria was given by authors Reinfeld and Hansen from principles stated by the National Institute of Management. Linear Programming may be used, according to this source, if: (1) the problem has a number of alternate choices, (2) differences in efficiencies exist among the choices, (3) the problem has definite upper and lower limits, (4) a definite goal has been stated, and (5) factors within the problem have interconnecting links and common units of measure.[48]

Quite obviously, there are minor differences of opinion among these and other sources of information concerning the requirements necessary to use linear programming.

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[47] Donald W. Moffett, "How You Can Use Linear Programming," Linear Programming and Inventory Management Seminar, Proceedings of seminar sponsored by Methods Engineering Council, Management Consultants, New York and Pittsburgh, September 15-16, 1955, p. M-2.

[48] N. V. Reinfeld and B. L. Hansen, "How You Can Use Linear Programming," Mill & Factory, 65 (December 1957), pp. 75-80.

[illegible]

Several points appear quite consistently throughout the various discussions in the literature, however. One of these is the necessity for a single objectively determined measure of effectiveness to be optimized in solving the problem. A study made for the United States Army delineated the problem in this manner:

> In any consideration of optimisation one of the first things which must be decided is what should be optimised. Very careful examination of this question is needed in each instance because if we are misled into maximising the wrong criterion our efforts may lead to waste rather than to savings. 49

Another necessary condition for application of linear programming is the presence of various alternative methods of using available resources. This is almost patently implicit in any problem-solving technique, for if there is only one way to accomplish some goal, the only decision to be made is whether or not to use the one available method. More unique to linear programming, however, is a corollary condition which requires the various factors and variables influencing a problem to be interdependent and interconnecting. If a man-hour is used in finishing a chair, for example, it cannot also be used for finishing a table, or for assembling a bookcase. Further, a desk cannot be finished unless it is assembled, which depends on completion of the cutting operation. Linear programming, then, is peculiarly adapted to solutions of <u>systems</u> of inter-related activities.

Finally, availability of data to permit the express-

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49 G. E. P. Box, "Some Considerations in Process Optimisation," <u>Technical Report No. 13</u>, Statistical Techniques Research Group, Section of Mathematical Statistics, Department of Mathematics, Princeton University, Princeton, New Jersey.

ion of relationships in mathematical terms is a condition which must be present in a problem if linear programming is to be employed. Synthesizing the requirements into a few words, it may be said that the linear programming method of planning may prove useful, if within a system of interrelated activities whose mutual dependencies may be expressed quantitatively, there exist alternative courses of action to pursue an overall, objectively measured goal of the system. It is significant to note that, in this pragmatic and operational definition of conditions necessary for linear programming applications, there is no mention of linearity. If the definition were theoretical, the condition of linearity could not be deleted. But, while the absence of linearity is not desirable, neither is it fatal in the practical world. If non-linearity is present in a problem, the solution may be found by making linear approximations, gathering various terms into one, neglecting non-significant quadratic or higher order expressions, or by a combination of these devices.[50] Another method of overcoming non-linearity is to introduce additional constraints into the problem to limit answers to relevant ranges where the functions are closely approximated by linear expressions.[51]

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[50] G. W. Evans, II, "A Transportation and Production Model," Naval Research Logistics Quarterly, 5 (June 1958),137.

[51] J. S. Aronofsky, "Linear Programming - A Problem-Solving Tool for Petroleum Industry Management," Journal of Petroleum Technology, XIV (December 1957), pp. 75-80.

Thus, although a literal translation of the theory of linear programming demands the condition of linearity, even theorists as highly regarded as Charnes and Cooper admit to the practicality of using shrewd approximations of non-linear functions.[52]

There is no single measure of effectiveness of the maintenance function available at this time. Because several criteria are involved in measuring maintenance performance, any attempts to derive an all-inclusive measure would have to include not only the objective of such a measure, but also the relationship of the various criteria to each other and the quantity and quality of information available on which to base the measuring device.[53] With the trend of management philosophy toward greater use of management science techniques, however, there is every indication that increased interest will eventually impel the derivation of a more complete maintenance measure. But until that degree of sophistication is reached, less perfect objective standards must be used while the presently unmeasurable aspects of maintenance continue to be evaluated by managerial judgement and experience.

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[52] A. Charnes and W. W. Cooper, Management Models and Industrial Applications of Linear Programming, Volume I (New York: John Wiley & Sons, Inc., 1961), p. 207.

[53] Herbert Terry, "Comparative Evaluation of Performance Using Multiple Criteria," Management Science, 9 (April 1963, 441.

One feasible, but highly imperfect criterion for measuring the maintenance function is the concept of opportunity cost. If, for example, an asphaltic concrete station road develops a minor failure, the estimator may conclude that it will cost $M in labor and materials to repair the failure immediately. It will cost $(M + N), however, to repair the same failure a year later if the work is postponed, because the condition will steadily worsen. Thus, the opportunity cost of choosing the alternative of delaying the repair work is $N.

This concept suffers from the same defect that impairs the usefulness of many other attempted definitions of maintenance performance, the measurement of input costs in lieu of output benefits. Also, it obviously neglects other costs and benefits, such as increased vehicle repair costs because of poor road conditions and the subjective utility derived by station forces from being associated with an activity which is maintained in good physical condition. Further, it implicitly assumes that the repair work will be performed at some time, whether in the present or in the future.

On the other hand, the opportunity cost concept does have certain advantages for the objective of this paper. It is quite apparent, for example, that maintenance costs vary with the age of structures, price increases for labor and materials, and the steady deterioration of certain maintenance conditions if repairs are delayed. Each of

these variables is a function of time, and the concept of opportunity cost includes this aspect. Further, the consideration of opportunity costs is common among maintenance managers when deciding what projects should be performed. Although probably not couched in terms of "opportunity cost", decisions to repair a particular structure instead of another are frequently made because costs of repair of the first will increase more rapidly than the second's if the work is postponed.

The purpose of this paper, however, is not the study of maintenance measurement. Therefore, opportunity cost has been chosen as a measure of effectiveness in describing one possible use of linear programming. The choice was made primarily because opportunity cost is an excellent illustrative device, common to the experience of many maintenance managers, and includes the aspect of profitability so well-known in the practical world. It is used in the example requiring a measure of maintenance effectiveness, therefore, with full knowledge of its severe limitations as an overall measure of efficiency, but with confidence that its proper role in the discussion will be clear.

One of the first possibilities which presents itself in considering uses of the linear programming tool is in planning maintenance projects to be accomplished by station forces during a forthcoming fiscal year. These projects could be to reduce a backlog of essential maint-

enance, or even those applying to the more routine jobs. On the surface this particular application appears to correspond nicely with the general conditions under which linear programming might be applied. There is an objective to be optimized, in this case the opportunity cost of projects under consideration. There are constraints on uses of available man-hour resources, in that the total amount of work which can be accomplished cannot exceed the total amount of man-hours available for the year. And there are alternatives, in the form of the various possible projects, to which the available man-hour resources may be applied. Letting C_i stand for the opportunity cost of delaying each maintenance project for a year, and X_i apply to the particular maintenance project under consideration ($i = 1,2,3,\ldots,n$) the objective function to be minimized would be:

(9) $Z = C_1X_1 + C_2X_2 + C_3X_3 + \ldots + C_nX_n.$

The constraining functions would apply to the total available man-hour resources in each maintenance shop during the year, or:

(10)
$$\begin{array}{l} A_{11}X_1 + A_{12}X_2 + A_{13}X_3 + \ldots + A_{1n}X_n \leq B_1 \\ A_{21}X_1 + A_{22}X_2 + A_{23}X_3 + \ldots + A_{2n}X_n \leq B_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ A_{m1}X_1 + A_{m2}X_2 + A_{m3}X_3 + \ldots + A_{mn}X_n \leq B_m \end{array},$$

where A_{ij} is the number of man-hours from shop i required to perform job j, and B_i is the total available man-hours

in shop i ($i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$). This appears to be a neat application of the linear programming method, but it is faulty in two respects. First, it includes no budgetary constraint to ensure that the total cost of the projects does not exceed the total yearly maintenance funding. This may be corrected by the addition of a constraint such as:

(11) $D_1X_1 + D_2X_2 + D_3X_3 + \ldots + D_nX_n \leq F$, where D_i is the total cost of each maintenance project ($i = 1,2,\ldots,n$), and F is the total funded budget figure. Another important constraint is also lacking, however, in that each maintenance project, X_i, is normally limited to values of either one or zero. In other words, the job either will or will not be accomplished. With this constraint, linear programming loses most of its value. It would be much easier and less time-consuming for the maintenance manager to merely list the projects being considered and plan for those which yield the greatest opportunity costs, until the shop capacities are reached. If the possibility of performing fractional parts of projects is admitted, however, a constraint to ensure that X_i will be limited to values of one or less may be inserted in the form:

(12)
$$
\begin{aligned}
X_1 &\leq 1 \\
X_2 &\leq 1 \\
&\ldots \\
X_n &\leq 1
\end{aligned}
$$

This requires a large simplex tableau and excessive computational time, and, again, the same results may be obtained merely by adding up jobs and parts of jobs which have the most favorable opportunity costs until the shop capacities are equaled. Therefore, this application of linear programming to planning finite numbers of specific maintenance projects is not feasible in that it requires unwieldy computations and the same answers may be obtained by much simpler methods.

A second application which might be considered is in planning for maintenance work which is of a repetitive nature. Examples of these are preventive maintenance inspections, care of lawns and grounds, maintenance of both vehicular and aircraft pavements, exterior structural painting, routine repairs to railroad and crane trackage, and other work of similar nature. In these cases it might be expected that certain minimum or specific amounts of work could be programmed for each year, with the objective of minimizing direct cost of the work. Maintenance projects in this category are more susceptible to measurement by various units, such as square feet of exterior painting, or square yards of runway and taxiway pavement maintenance. Again, the limitations imposed are shop capacities and budgetary considerations. The objective would be to minimize direct, not opportunity, costs, in the form:

(13) $Z = C_1X_1 + C_2X_2 + C_3X_3 + . . . + C_nX_n$,

[illegible]

subject to the shop capacity constraints:

$$(14) \quad \begin{aligned} &A_{11}X_1 + A_{12}X_2 + A_{13}X_3 + \ldots + A_{1n}X_n \leq B_1 \\ &A_{21}X_1 + A_{22}X_2 + A_{23}X_3 + \ldots + A_{2n}X_n \leq B_2 \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &A_{m1}X_1 + A_{m2}X_2 + A_{m3}X_3 + \ldots + A_{mn}X_n \leq B_m \end{aligned}$$

the requirements that a number of units of each type of work are to be performed:

$$(15) \quad \begin{aligned} &X_1 = D_1 \\ &X_2 = D_2 \\ &\ldots\ldots \\ &X_n = D_n \end{aligned}$$

and the budgetary constraint:

$$(16) \quad C_1X_1 + C_2X_2 + C_3X_3 + \ldots + C_nX_n \leq F.$$

In this example, A_{ij} represents the number of man-hours of shop i required per unit of maintenance work of type j, B_i is the capacity restriction, in man-hours, of the ith shop, D_j is the number of units of work type j required per year, C_j is the total unit cost of the jth type of work, and F is the annual budgetary limit ($i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$).

While the simplex tableau would be quite large because of the additional budgetary and work load constraints, equations (15) and (16), the number of different types of maintenance work will probably be less than the number

of maintenance projects under consideration in the first example. Thus, the implicit value, or economic worth, information derived from the model may be sufficiently valuable as to warrant the computational effort. If computer time is available, of course, the computational effort no longer is a problem, and the question to be answered then is whether or not the resulting information is worth the cost of the computer program. As for the direct answer as to the resources to utilize in performing the maintenance tasks, however, this example of linear programming is as valueless as the first. With the constraint imposed that certain amounts of effort <u>must</u> be expended on each type of work, equations (15), the solution to the minimum cost problem is simply those minimum quantities of effort times the unit cost of the effort. Therefore, unless the maintenance manager wishes to obtain the implicit values of his manpower resources, linear programming in this type of application is not worthwhile.

The difficulty encountered in applying the technique in the first two examples appears to be lack of true alternative courses of action. Maintenance work is somewhat unique in that the normal decision is often not how but whether or not to accomplish certain projects. Linear programming, however, has probably found its most beneficial applications in situations in which the critical question is how to perform a given task. This is true

in many of the examples given in texts, and is certainly true in the case of perhaps the leading American industrial user of linear programming, the petroleum industry. In the latter instance, the technique has proven invaluable in answering the question of how to blend various inputs to produce a mixture of petroleum products for maximum profits. The same type of alternatives, however, was not present in the examples considered so far. In other words, the problems were too restrictive for beneficial application of linear programming. But to imply that there are no alternative methods of accomplishing a particular type of maintenance project, that there is but one way to perform a certain job, would be incorrect.

Consider the simple example of mowing grass at a naval station. There are various methods of accomplishing this work, depending upon the type of equipment used. Each of the various methods can accomplish the function at different costs per square yard of lawn, or per acre of land adjacent to runways. Even without the factor of different materials which could be used in performing structural maintenance, for example, it can be seen that <u>how to</u> accomplish a particular job may be a very relevant question. Assume, as an illustration, that there are repetitive maintenance projects under planning consideration. For each of the projects there are different combinations of men, material, and machinery (from hand tools through heavy construction equipment) which may be used to

accomplish the work. In addition, there is the very real possibility of awarding a contract for all or some of the work to private contractors. In this situation there are many alternative uses of available resources in the numerous combinations of men, materials, machinery, and money to accomplish the maintenance objective at, let us say, minimum direct cost.

For purposes of facilitating the design of the model for this problem, let each of the various ways in which a particular job may be performed be designated as a separate entity. If there are "n" different methods of painting the exterior of buildings, for example, these can be looked on as "n" separate types of work, $X_{11}, X_{12}, \ldots, X_{1n}$. Letting C_{ij} stand for the unit cost of performing the ith job in the jth manner ($i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$), the objective would be to minimize total cost:

$$(17) \quad Z = C_{11}X_{11} + C_{12}X_{12} + \ldots + C_{ij}X_{ij} + \ldots + C_{mn}X_{mn}$$

The following constraining functions would describe the limitations of shop capacity:

$$(18) \quad \begin{array}{l} A_{111}X_{11} + A_{121}X_{12} + \ldots + A_{211}X_{21} + \ldots + A_{mn1}X_{mn} \leq B_1 \\ A_{112}X_{11} + A_{122}X_{12} + \ldots + A_{212}X_{21} + \ldots + A_{mn2}X_{mn} \leq B_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ A_{11q}X_{11} + A_{12q}X_{12} + \ldots + A_{21q}X_{21} + \ldots + A_{mnq}X_{mn} \leq B_q, \end{array}$$

in which A_{ijk} stands for the man-hours of the kth shop required per unit of the jth method of accomplishing the

[illegible]

ith type of work, and B_k signifies the total available man-hours in the kth shop ($k = 1, 2, \ldots, q$). Additional constraints would denote the necessity for the summation of the number of units of various methods of doing a job to equal the total number of units of that type of work to be performed. For example, if there were a total of 250 acres of improved grounds adjacent to runways to be mowed, the number of acres mowed by the first method, plus the number mowed by the second method, etc., would have to equal 250. In general mathematical notation:

$$
\begin{aligned}
(19)\quad & X_{11} + X_{12} + X_{13} + \ldots + X_{1n} = D_1 \\
& X_{21} + X_{22} + X_{23} + \ldots + X_{2n} = D_2 \\
& \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
& X_{m1} + X_{m2} + X_{m3} + \ldots + X_{mn} = D_m,
\end{aligned}
$$

in which D_i is the total number of units of the ith job ($i = 1, 2, \ldots, m$) to be performed.

A further constraint to be stated explicity would be the amount of work which could be performed by contract. Even if this "limitation" were infinity, a contractor might accept unlimited quantities of work, it would be incorrect to leave out the expressions showing this relationship. Assuming that, in coding the combinations of jobs and methods, the numeral nine always stands for contractor work, these constraints would be:

$$(20)\quad \begin{aligned} &X_{19} \leq E_1 \\ &X_{29} \leq E_2 \\ &\cdot\ \cdot\ \cdot\ \cdot \\ &X_{mq} \leq E_m \end{aligned}$$

where X_{iq} specifies the number of units of the ith work to be performed by contract, and E_i represents the contractors' capacities for that type of work ($i = 1, 2, \ldots, m$).

Two final constraints assure that all limitations have been considered. The first defines the restrictions on availability of critical machinery, equipment, and special tools. Letting G_{ijp} imply the number of hours of the pth type of machinery, equipment, or tools required per unit of the ith method of performing the jth type of work, and H_p stand for the total number of hours available for using the particular piece of machinery of type p ($p = 1, 2, \ldots, v$), the constraints may be written as:

$$(21)\quad \begin{aligned} &G_{111}X_{11} + G_{121}X_{12} + \cdots + G_{ij1}X_{ij} \leq H_1 \\ &G_{112}X_{11} + G_{122}X_{12} + \cdots + G_{ij2}X_{ij} \leq H_2 \\ &\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ &G_{11v}X_{11} + G_{12v}X_{12} + \cdots + G_{mnv}X_{mn} \leq H_v \end{aligned}$$

The final constraint would ensure that budgetary allocations are not exceeded. Letting F represent the total maintenance budget under consideration, while the other symbols retain their previous definitions:

$$(22)\quad C_{11}X_{11} + C_{12}X_{12} + \cdots + C_{ij}X_{ij} + \cdots + C_{mn}X_{mn} \leq F$$

The least cost model thus developed would require considerable analysis of the maintenance function, but it would result in a more penetrating managerial comprehension of the functions of Public Works Department at a shore activity. It also appears to embody the other advantages of the linear programming method previously mentioned; i.e., answers to the direct question of resource allocation, full knowledge of the implicit value, or economic worth in terms of maintenance cost, of the various resources, the capability of letting management experiment on paper with various policies and procedures, and the freeing of the manager's energies for broader considerations after the model is in operation. It is limited, however, to cases of repetitive work; that is, types of work which recur in identical form in repetitive instances. Further, there is a very real question as to the limitations of the resources involved. Theoretically, it may answer the question of how many man-hours should be devoted to repetitive work, but it ignores the question of how best to utilize the remainder of the work force in work which is of a non-repetitive nature. Further, once the least-cost methods of accomplishing the various classes of repetitive work are established, and this could possibly be done with less time-consuming analysis than linear programming, it loses its usefulness until changed conditions require a new analysis. If certain equipment is "scarce" in terms of limited available capacity, for example, the

most logical solution would be to obtain the required capacity. A final difficulty stems from the fact that this model does not consider the entire work force. If it were adapted to all maintenance work, a set of constraints would be required to ensure that all the work force is fully employed, that is, all shop capacity is utilized, to secure the benefits of level employment.

Many of these problems might be overcome if, instead of attempting to minimize cost, the objective were to maximize maintenance output. Consider, for a moment, a fictitious unit of output, called "maint" for mnemonic purposes. If the accurate definition of this unit could be derived, the objective of maximum output might be stated easily in terms of the general linear programming problem. With relatively minor difficulty, problems encountered in the previous least-cost model might be neutralized, and the method could provide a versatile and valuable planning tool. Unfortunately, an overall, accurate measurement of maintenance output is not yet available.

IV

SUMMARY AND CONCLUSIONS

Summary

The practice of maintenance management in the Navy seems to be more advanced than the same function within civilian industrial firms. Even in the Navy, however, the overall difficulty of managing the shore facility maintenance function, and the potential benefits which might accrue almost force a consideration of the relatively young management science discipline. As a method of applying scientific logic and quantitative analysis to managerial decision problems, management science provides opportunities to increase the scope and intensity of managerial information necessary for decision-making. It does not threaten to replace the manager's judgement or wisdom with elegant analyses and solutions of problems. Rather, it attempts to supplement those necessary managerial skills with increased comprehension and reasoning abilities.

Among the many management science techniques, linear programming stands out as a particularly forceful and versatile tool. Not only does it answer the direct question of how to apply scarce resources to alternative opportunities to optimize a specific objective, but it has several other decidedly beneficial aspects. One of the more important is the capability of providing managers

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with the economic cost of constraints measured in terms of the overall objective of the organization. Further, it provides a method by which contemplated plans, policies, and procedures may be tested for their effect on the goals before they are implemented. Increased managerial understanding of the variables of the organization, and the unfettering of executive energies from routine details are additional returns from appropriate linear programming applications. Because of these substantial potential advantages, an inquiry into possible uses of the method in maintenance management of the Navy's shore facilities was considered desirable.

In conducting the investigation, several assumptions were made to facilitate the theoretical nature of the study. The supposition of linearity in various aspects of the maintenance function was considered a reasonable approximation to actual situations. It was also assumed that there is an opportunity cost pay-off in performing maintenance projects in the present in lieu of delaying them to a future date, and that this pay-off may be measured by means of reasonably accurate cost estimates.

Because of the necessity to rather severely define the scope of the investigation, several limitations were introduced. The first is simply the lack of practical testing of hypotheses under actual field conditions. The second is more of a danger of misinterpretation than a constraint on the inquiry. There

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was no attempt to conduct a thorough investigation of a maintenance problem through management science procedures. Instead, the paper presents merely a preliminary and limited probe into possible uses of a particular technique, linear programming.

The third limitation is the assumption that opportunity costs of maintenance projects are a sufficiently precise measure of maintenance productivity for purposes of this study. In attempting to obtain an objective measure of maintenance, <u>the opportunity cost concept was chosen as sufficiently reasonable only for purposes of illustrating a possible linear programming application</u>. The concept not only disregards many returns on the maintenance effort, but it also may be inapplicable, except for long range planning, in many routine maintenance situations where costs increase only slightly, if at all, with short periods of time. Alternatives for measuring the maintenance effort for the study range from a vague but all-inclusive gauge such as "utiles", to the concrete but very limited concept of opportunity cost. The latter was chosen merely as a more efficient illustrative device, not to represent it as a valid measure of maintenance effectiveness.

In reviewing selected literature of the management science and maintenance management fields, the lack of studies directed at attempts to apply linear programming to facility maintenance problems was strikingly demonstrated.

Equally conspicuous, however, were attempts on the part of industrial managers to define maintenance performance.

Conclusions

The principle problem encountered in attempting to discover suitable applications of the technique was that of finding truly alternative uses of scarce resources. In many instances in the maintenance management area, the least costly method of accomplishing a given task may be obtained from less extensive and time-consuming analyses than linear programming. A least-cost model for analysis of maintenance work of repetitive nature was developed to include the possible alternative of awarding contracts to private firms for portions of the maintenance effort. Field application is required to actually test the model, but it is likely that it would prove to be of limited value in that it considers only a portion of the maintenance work force.

Implications of the Study

One of the major problems encountered in investigating possible applications of linear programming lay in the practical necessity of maintaining a level maintenance work force at full employment. In order to secure the potential benefits of the linear programming technique with the economies of a level work force, an objective of maximizing total output from the work force is more feasible than attempting to minimize cost. This requires,

however, a quantitative measure of maintenance output. If such a measure were available, the practice of maintenance management could be advanced by several strides in the direction of making "better" and, eventually, "best" decisions through the application of the management science philosophy.

Recommendations for Further Study

One of the most pressing needs of the maintenance manager is the ability to measure and define the output of the maintenance effort. Even in the private business world, with the ultimate criterion of profit available as an objective measurement, much of the energy devoted to this general area has been in measuring inputs to the maintenance function. Such devices as costs per dollar of plant value serve to weigh the budget of the maintenance department, but do little towards determining how large a contribution is made to the organization's overall objective.

In the Navy, without the advantage of a profit yardstick, the difficulty of objectively measuring the maintenance output is even more perplexing. The importance of such an objective measurement, however is immediately apparent if maintenance management is to operate with increasing efficiency and effectiveness. Considerable work has been done in the theory of general utility

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measurement and the "military essentiality" concept of quantifying subjective decisions.[54] Since there is much of the maintenance effort which results in subjective utility, it would appear that the theory of utility measurement may yield substantial benefits in attempting to accurately define maintenance effectiveness. Since future progress in maintenance management may hinge on such an accurate definition, the attempt should be made.

In addition to linear programming, other management science techniques offer decided promise to the maintenance manager. Replacement theory, for example, may be of benefit in attempting to find the optimal point in time at which to replace an existing structure. Queing theory, as well, might be able to shed light on the level of maintenance forces to be combined with acceptable delay times while awaiting work to be accomplished, for overall optimization of station effort. These are but two of the many examples in which the management science approach promises substantial benefit to maintenance managers. The Navy has long enjoyed the forward-looking philosophy of its facility maintenance organization. A trend towards increasing applications of the modern management techniques of management science will assure the continuance of this progressive outlook.

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[54] W. G. Mellon, "A Selected Descriptive Bibliography of References in Priority Systems and Related Non-price Allocators," Naval Research Logistics Quarterly, 5 (March 1958), pp. 17-27.

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